Correlation scenarios and correlation stress testing

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Overview

- **Correlation** lies at the heart of many financial applications: portfolio risk-management, diversification, hedging.
- Principal idea: link **economically meaningful scenarios** to **correlation scenarios**
- First paper **"London Whale"**:

  Packham, N. and Woebbeking, F.: *A factor-model approach for correlation scenarios and correlation stress-testing*. Journal of Banking and Finance, 101 (2019), 92-103. link

Overview

- Current **working paper**:

  Packham, N. and Woebbeking, F.: *Correlation scenarios and correlation stress testing* . link

- Objectives:
  - **Correlation factor model** for any kind of financial asset portfolio
  - **Bayesian factor selection** to incorporate a priori knowledge
  - **Stress testing**: portfolio effect of adverse correlation scenarios
  - **Reverse stress testing**: identify extreme yet plausible scenarios

The "London Whale"

- **"London Whale"**: 2012 Loss at JPMorgan Chase & Co. of approx. 6.2 bn USD on a credit derivatives portfolio
- **Authorised trading position**, hence risk management problem
- **Synthetic credit portfolio (SCP)**: 120 long and short positions, **CDX** and **iTraxx** index + tranche products, investment grade and high-yield
- **"Smart short" strategy**: credit protection on high yield is financed by selling protection on investment grade indices.
- Timeline:
  - End of 2011: decision to reduce SCP's risk-weighted assets (RWA's).
  - Avoid liquidation costs by **increasing** positions with opposite market sensitivity (hedges).
  - 23 March 2012: Senior executives ordered to stop trading on SCP; net notional of 157 bn USD (up 260% from September 2011).
- **Risk management** of SCP focussed on **value-at-risk (VaR)** and **CSW-10** (credit spread widening of 10 basis points).
- Publicly available information: JPMorgan, 2013; United-States-Senate, 2013a,b

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Overview

Interest-rate modelling: Correlation parameterisation

- Parametric correlation models widespread in

  interest-rate modelling / LIBOR market model,

  e.g. Rebonato (2002); Brigo (2002); Schoenmakers and Coffey (2000); Packham (2005)

- Simplest case: Correlation c_{ij} between two forward LIBOR's is given by

$$c_{ij} = e^{-\beta|i-j|},$$

  where $\beta > 0$ is a parameter, and i, j represent maturities.

- Captures stylised fact that **correlations decay with increasing maturity difference**

Link correlations to risk factors

- Idea: Carry over **"distance" measure** to other **risk factors**, such as geographic regions, industries, investment grade vs. high-yield, ...
- Association of asset $i \in \{1, \ldots, p\}$ with factor $k \in \{1, \ldots, d\}$:

$$\mathbf{1}_{\{k,i\}}$$

[Assume this as given for the time being.]

- Correlation parameterisation:

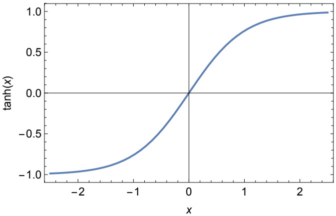
$$c_{ij} = \tanh\Bigg(\underbrace{\sum_{k=1}^{d} \lambda_k |\mathbf{1}_{\{k,i\}} - \mathbf{1}_{\{k,j\}}|}_{\text{"inter"-correlations}} + \underbrace{\sum_{k=1}^{d} \nu_k \mathbf{1}_{\{k,i\}} \mathbf{1}_{\{k,j\}}}_{\text{"intra"-correlations}} \Bigg),$$

with coefficients $\lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d \in \mathbb{R}$.

Link correlations to risk factors

- $\tanh : \mathbb{R} \to [-1, 1]$ allows for negative correlations.
- $\tanh$ used in inferential statistics on sample correlation coefficients ($\rightsquigarrow$ Fisher transformation).
- The following summation formula is helpful for a rough interpretation of the coefficients:

$$\tanh(x + y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$$



Correlation parameterisation

- Given a sample correlation matrix at one time point, the coefficients $\lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d$ can be determined e.g. by **ordinary least squares** on $\text{arctanh}(c_{ij})$, the inverse of $\tanh$.
- Simple correlation **scenarios** such as "the correlation between assets exposed to factor k and assets not exposed to factor k increases" is then implemented by increasing λ_k (e.g. Europe vs US).
- Likewise, a scenario such as "the correlation of firms exposed to factor k increases" is implemented by increasing ν_k (e.g. within Europe).
- With parameters calibrated on a regular basis, the parameter history can be used to **obtain realistic scenarios** (reverse stress test).

Overview

Motivation

Methodology

Correlation parameterisation

Factor selection

Stress testing

Application (equity portfolio)

Conclusion

Principal ideas

- Risk factors in "London Whale" were tailored to specific portfolio.
- In practice, factor models use **industries** and **countries** as factors to model asset correlations.
- Problem: **How to assign factors to assets?** link
- Number of factors should be **small**, but include all **important** factors.
- **Prior information**: country of firm's headquarter, primary industry
- $\rightsquigarrow$ **Bayesian variable selection** to determine small number of factors driving asset return

Bayesian variable selection

- Different methods, e.g.
  - **Bayesian model selection** compares **posterior probabilities** of different models.
  - **Spike and slab priors** include an indicator variable for each coefficient and determines the indicator variable's **posterior probability** of taking value one.
- In our setting, **Bayesian model selection** worked best.

Bayesian model selection

- Denote candidate models by M_i, $i = 1, \ldots, m$.
- In a linear regression setting, each model M_i includes a specific subset of independent variables ($=$ potential risk factors) and excludes the other variables.
- **Posterior model probability**:

$$p(M_i|\boldsymbol{y}) \propto p(\boldsymbol{y}|M_i)p(M_i),$$

where

- $\boldsymbol{y}$ is the time series of a **firm's asset returns**,
- $p(M_i)$ is the **prior model probability**,
- $p(\boldsymbol{y}|M_i)$ is called the **marginal likelihood**.

(see e.g. Appendix B.5.4 of (Fahrmeir *et al.*, 2013))

Bayesian model comparison

- **Posterior inclusion probabilities (PIP)**:

$$\mathbf{P}(\mathbf{1}_{\{\beta_k \neq 0\}} = 1|\boldsymbol{y}) = \sum_{\beta_k \in M_i} \mathbf{P}(M_i|\boldsymbol{y}).$$

- If number of parameters p is large, then full calculation of 2^p posterior model probabilities is infeasible.
- $\Rightarrow$ Use **Markov Chain Monte Carlo (MCMC)** simulation.
- Factors with PIP greater 0.5 are selected

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Stress-testing correlations

- **Stress-test**: Effect on portfolio due to an adverse scenario.
- A shift in correlation has no *instantaneous* effect on portfolio value, therefore consider **portfolio risk**.
- Portfolio risk measured by **value-at-risk (VaR)** in variance-covariance approach:

$$\mathrm{VaR}_\alpha = -V_0 \cdot \mathrm{N}_{1-\alpha} \cdot \left(\mathbf{w}^\intercal \, \boldsymbol{\Sigma} \, \mathbf{w}\right)^{1/2},$$

  with
  - current position value V_0,
  - $\mathrm{N}_{1-\alpha}$: $(1-\alpha)$-quantile of the standard normal distribution,
  - vector of portfolio weights $\mathbf{w}$ and
  - covariance matrix $\boldsymbol{\Sigma}$.
- For **correlation stress test**, only need to consider portfolio variance

$$\mathbf{w}^\intercal \, \boldsymbol{\Sigma} \, \mathbf{w}.$$

Revere stress testing

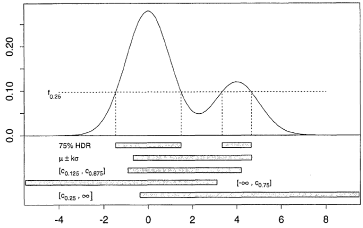
- What is the **worst scenario** amongst all scenarios that occur within some pre-given **range**?
- Restrict **risk-factor distribution** $(\lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d)$
- Univariate setting: quantile
- Multivariate setting:
  - Mahalanobis distance (Mahalanobis, 1936),
  - highest density regions (HDR) (Hyndman, 1996a),
  - concepts based on norms, e.g.(Serfling, 2002).
- Maha is closely tied to the normal or to elliptical distributions.
- HDR allows for more flexibility (e.g. skewness and tail heaviness).

Highest density region (HDR)

- Let $f(x)$ be the density function of a random vector X
- The $100(1-q)\%$ HDR is the subset of $R(f_q)$ of the sample space of X such that

$$R(f_q) = \{x : f(x) \geq f_q\}$$

where f_q is the largest constant such that $\mathbf{P}(X \in R(f_q)) \geq 1 - q$.



(Hyndman, 1996b)

- **Worst-case** scenario within given HDR:

$$\boldsymbol{\beta}^* = \underset{\{\boldsymbol{\beta} \in R(f_q)\}}{\text{argmax}} \ \text{VaR}_\alpha(\boldsymbol{\beta}).$$

Overview

Motivation

Methodology

Application (equity portfolio)

Factor selection and fit

Stress test

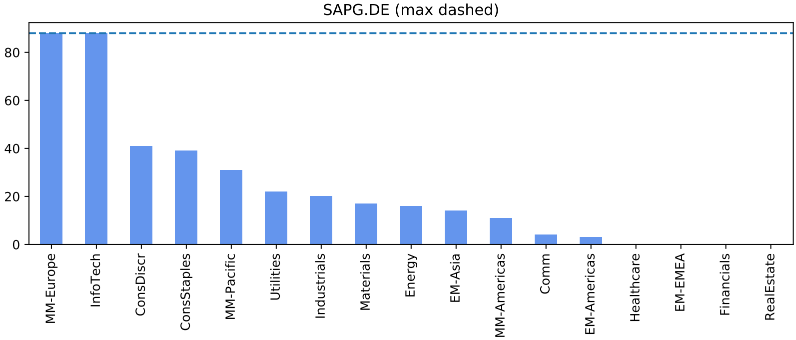
Conclusion

Factor selection

- Factors: MSCI stock indices representing 6 geographic regions and 11 industries
- Individual stocks: 505 S&P constituents, 30 DAX constituents
- Daily data from 1999-Jan 2021 (Source: Bloomberg, MSCI, Reuters)
- Factor assignment re-calibrated every quarter, based on 3-years of daily data (88 quarters)
- Prior: hard-code primary geographic region and industry,
- 6 factors on expectation

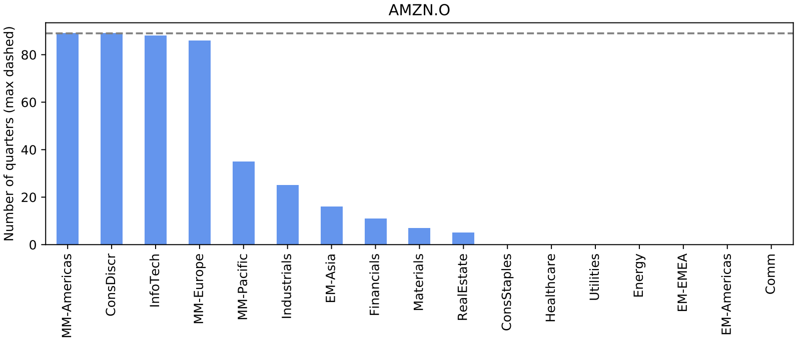
Factor selection

- Number of quarters that each factor is included for SAP
- German IT company

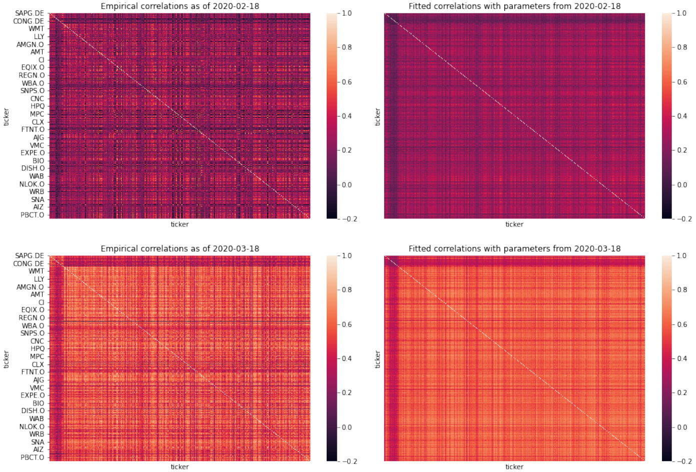


Factor selection

- Number of quarters that each factor is included for Amazon:
- US based online retailer with strong presence in Europe
- World's largest provider of computing services (AWS)

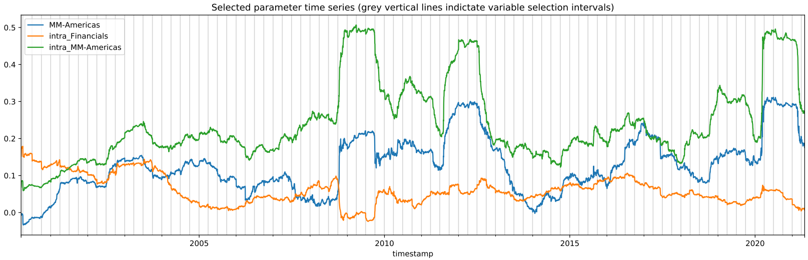


Correlations at beginning of Covid-19 pandemic



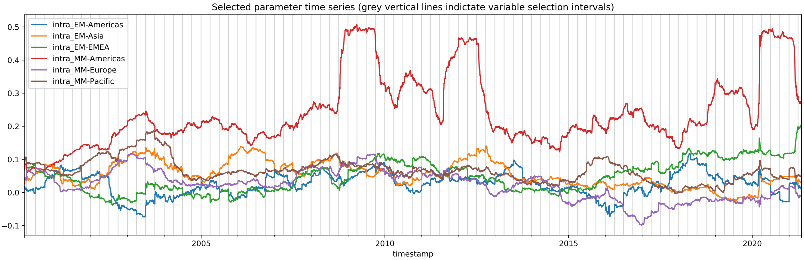
- Empirical & fitted correlations; top: 18 Feb, bottom: 18 Mar 2020.

Factor coefficients



- Fitted parameters for risk factors with high loads.

Factor coefficients



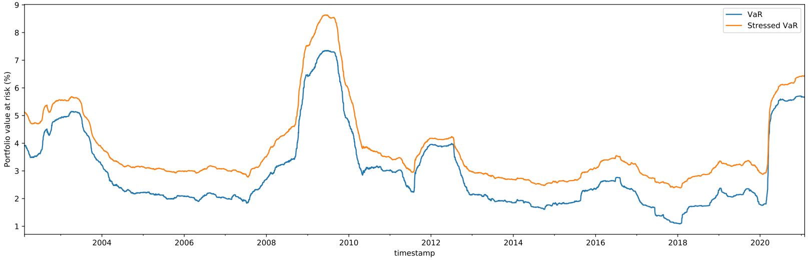
- Fitted “intra” parameters for selected risk factors (“ν_k”)

Overview

Risk-factor distribution

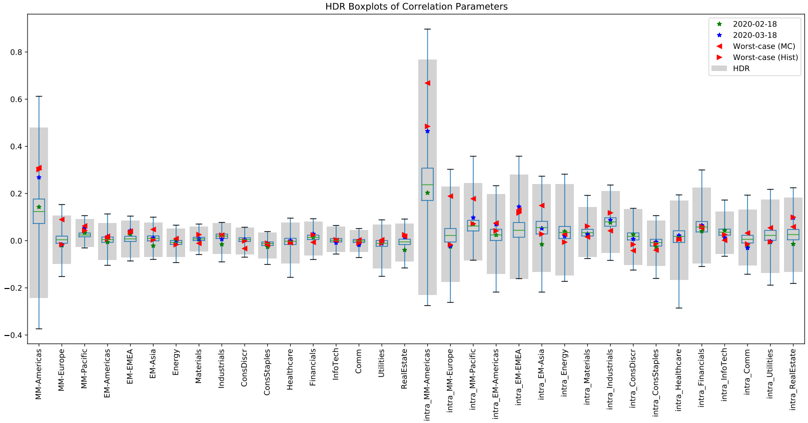
- Fit time series of risk factor parameters $(\lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d)$ to **Normal-Inverse Gaussian (NIG)** distribution
- NIG: generalisation of normal dist. that allows for skewness and higher variation in tails
- Calibration via using expectation-maximization (EM) algorithm, (McNeil *et al.*, 2005, Chapter 3) and Dempster *et al.* (1977)

Value-at-risk impact



- Blue: $\text{VaR}_{99\%,1 \text{ day}}$ on equally-weighted portfolio of DAX and S&P 500
- Orange: Stressed $\text{VaR}_{99\%,1 \text{ day}}$ on reverse stress scenario of 5 April 2021.

Reverse stress testing (Covid-19 pandemic)



- Worst-case scenario within 95% HDR (18 Feb 2020)
- Triangles: worst-case scenarios (MC sim., Hist. sim.)
- Stars: Scenarios on 18 Feb (green) and 18 March (blue)

Conclusion

- We develop a correlation stress testing framework, linking risk factors with correlations.
- Risk factors (e.g. industries, countries) are linked firms via Bayesian variable selection methods.
- Reverse stress tests are conducted by assigning the factor loadings a distribution and determining the worst-case scenario within a HDR.

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Thank you!

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