



School of  
Management and Law

## Interpretable Machine Learning for Diversified Portfolio Construction

Publication available at JFDS: <https://doi.org/10.3905/jfds.2021.1.066>

Markus Jaeger, Munich Re Markets

Stephan Krügel, Munich Re Markets

Dimitri Marinelli, Munich Re Markets and Marie Skłodowska-Curie Individual Fellowship

Jochen Papenbrock, NVIDIA GmbH

Peter Schwendner, Institute of Wealth & Asset Management, Zurich University of Applied Sciences

6<sup>th</sup> European Conference on Artificial Intelligence in Industry and Finance

Winterthur 9.9.2021



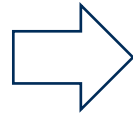
ARTIFICIAL INTELLIGENCE IN FINANCE  
KNOWLEDGE EXCHANGE PLATFORM  
Fin – Tech HO2020 project



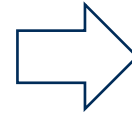
This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No.750961

# ML-based concept for benchmarking robustness of portfolio strategies

Synthetic  
market data  
generator



Backtesting  
systematic  
portfolio strategy



Explain performance  
by market data  
features

For return time series:  
Bootstrapping, GANs  
For correlation matrices:  
«Matrix Evolutions», CorrGAN

Alternative systematic portfolio  
strategies:  
Equal risk contribution (ERC)  
Hierarchical Risk Parity (HRP)

Explainable machine learning (XAI):  
Train explanation model (XGBoost)  
Discuss Shapley values (TreeSHAP)

# Risk-Controlled Diversified Portfolios: The Risk Parity idea

Risk parity idea:

- Long positions in the most liquid futures markets across asset classes.
- Lever up low-risk assets to achieve similar risk contributions across asset classes.
- Rebalance regularly, i.e. monthly or quarterly.
- Control risk at portfolio level using dynamic leverage («volatility target»).

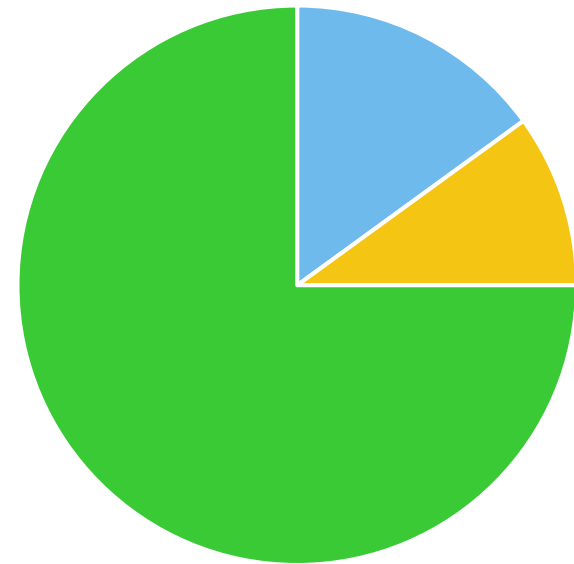
# Risk Parity idea: an Example

Equal Risk Contributions



■ Equities ■ Commodities ■ Fixed Income

Fixed Income dominates  
Market Value Allocation



■ Equities ■ Commodities ■ Fixed Income

Fixed income has low volatility, but it is levered up to achieve a similar risk contribution as equities and commodities.

Negative correlations from fixed income to other asset classes allows a higher portfolio leverage.

# March 2020: equities and bonds show a correlated correction, causing losses for risk parity funds.

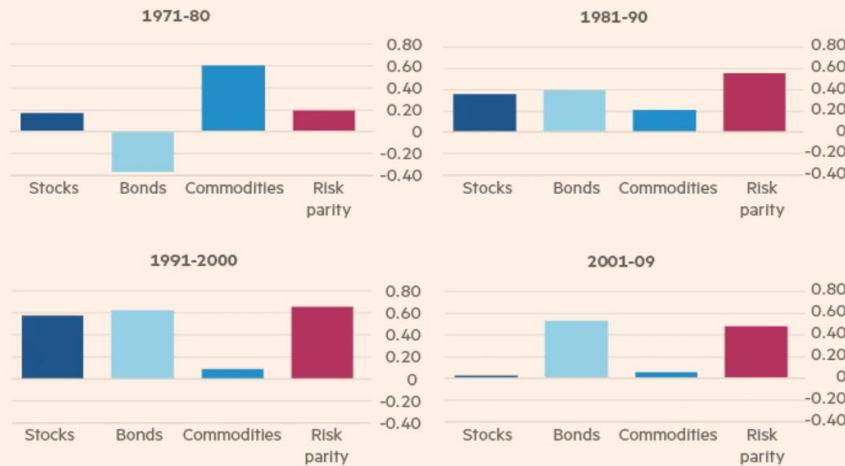
## Analysts point finger at 'risk parity' strategy in market rout

Funds are designed to perform well at all times but some say they exacerbate sell-offs

Robin Wigglesworth MARCH 20 2020

A simple risk parity strategy has historically delivered better, steadier returns than most individual asset classes

Sharpe ratio



Source: AOR © FT

## Treasuries have sold off in tandem with stocks lately, pushing yields higher



Source: Refinitiv © FT

## Risk parity funds pumelled by bond and stock sell-off

S&P index showing performance of typical risk parity fund with a 10% volatility target



Source: Refinitiv © FT

Ref: Danilo Vassallo, Lieven Hermans and Thomas Kostka, "Volatility-targeting strategies and the market sell-off", Financial Stability Review, May 2020 [https://www.ecb.europa.eu/pub/financial-stability/fsr/focus/2020/html/ecb.fsrbox202005\\_02-f6616db9be.en.html](https://www.ecb.europa.eu/pub/financial-stability/fsr/focus/2020/html/ecb.fsrbox202005_02-f6616db9be.en.html)

# Can HRP achieve more robust portfolios despite unstable correlations?

## ERC

Equal Risk Contribution

Distributes the risk evenly  
among the assets.

## HRP

Hierarchical Risk Parity

Uses ML  
(Hierarchical Clustering)  
for asset allocation

- Volatility target 5%
- Monthly rebalanced portfolio
- Half-turn transaction cost of 2 bp (flat)
- 17 futures markets (equities, commodities, fixed income)
- 20 years of data 5/2000 – 6/2020

## Equal Risk Contribution (ERC)

The risk contribution of the  $i$ -th asset in the portfolio is

$$\mathcal{RC}_i = \frac{w_i [\Sigma w]_i}{\sqrt{(w' \Sigma w)}}$$

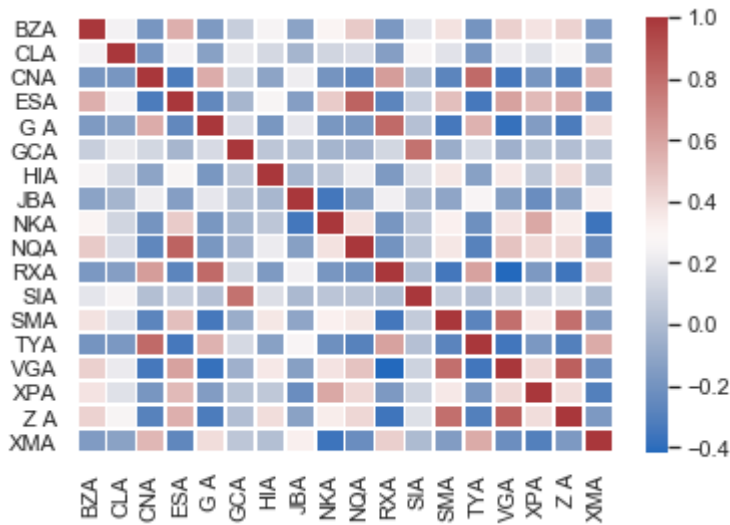
where  $\Sigma$  is the VCV matrix, and  $w$  the portfolio weights.

The ERC portfolio weights are the solution of the optimization

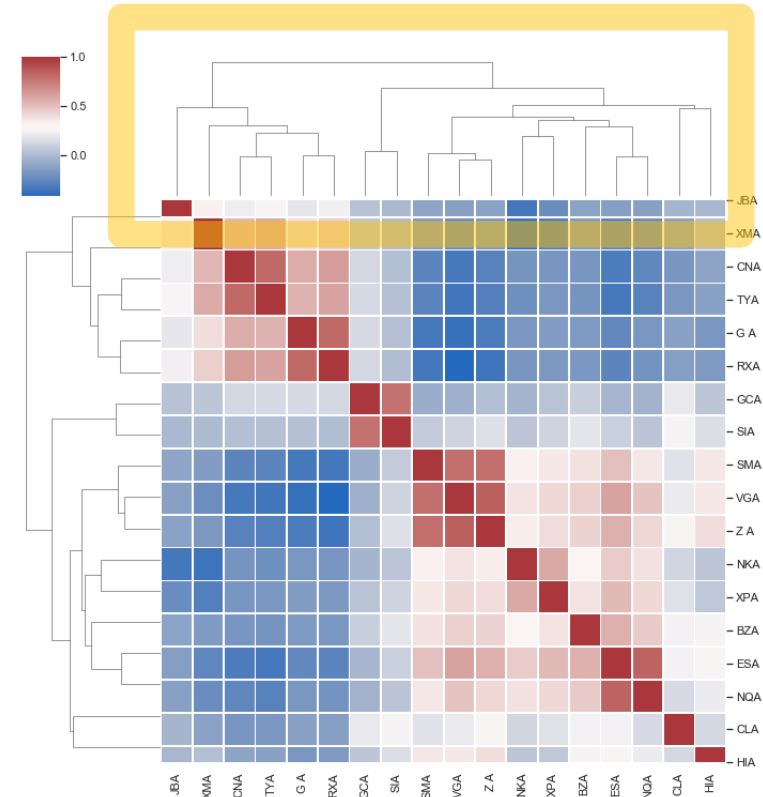
$$\operatorname{argmin}_w \left[ \sum_{i=1}^N \left( \frac{\mathcal{RC}_i}{\sqrt{(w' \Sigma w)}} - \frac{1}{N} \right)^2 \right].$$

“minimize deviations from equal risk contributions across assets”

# Hierarchical Risk Parity (HRP) – first step



Clustering

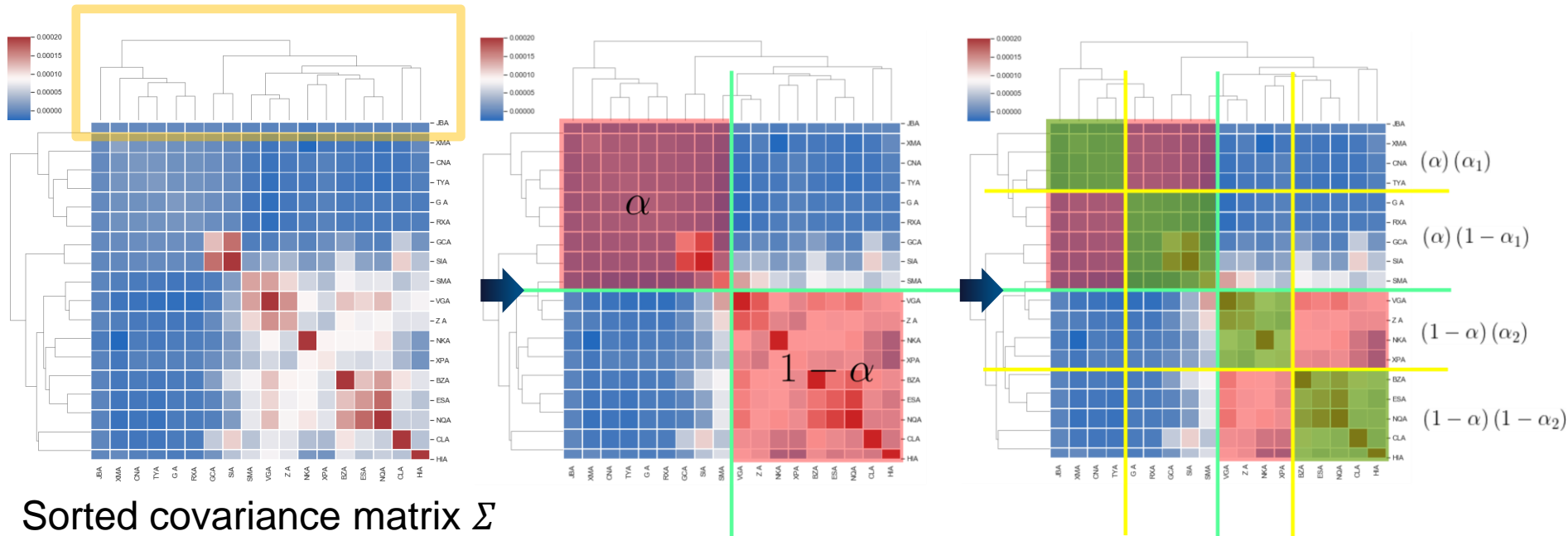


Correlation matrix gets sorted by similarity  
(Gowers + Euclidean distance)



# Hierarchical Risk Parity (HRP) – second step

Recursive bisection of the covariance (not correlation!) matrix



$$\alpha = 1 - \frac{\sigma^2(w^{(1)})}{\sigma^2(w^{(1)}) + \sigma^2(w^{(2)})}$$

$$\sigma^2(w^{(j)}) = w^{(j)T} \Sigma^{(j)} w^{(j)} \text{ and } w^{(j)} = \frac{1/\text{diag}[\Sigma^{(j)}]}{\text{tr}(\text{diag}[\Sigma^{(j)}]^{-1})}$$

# (CAL)MAR Ratio

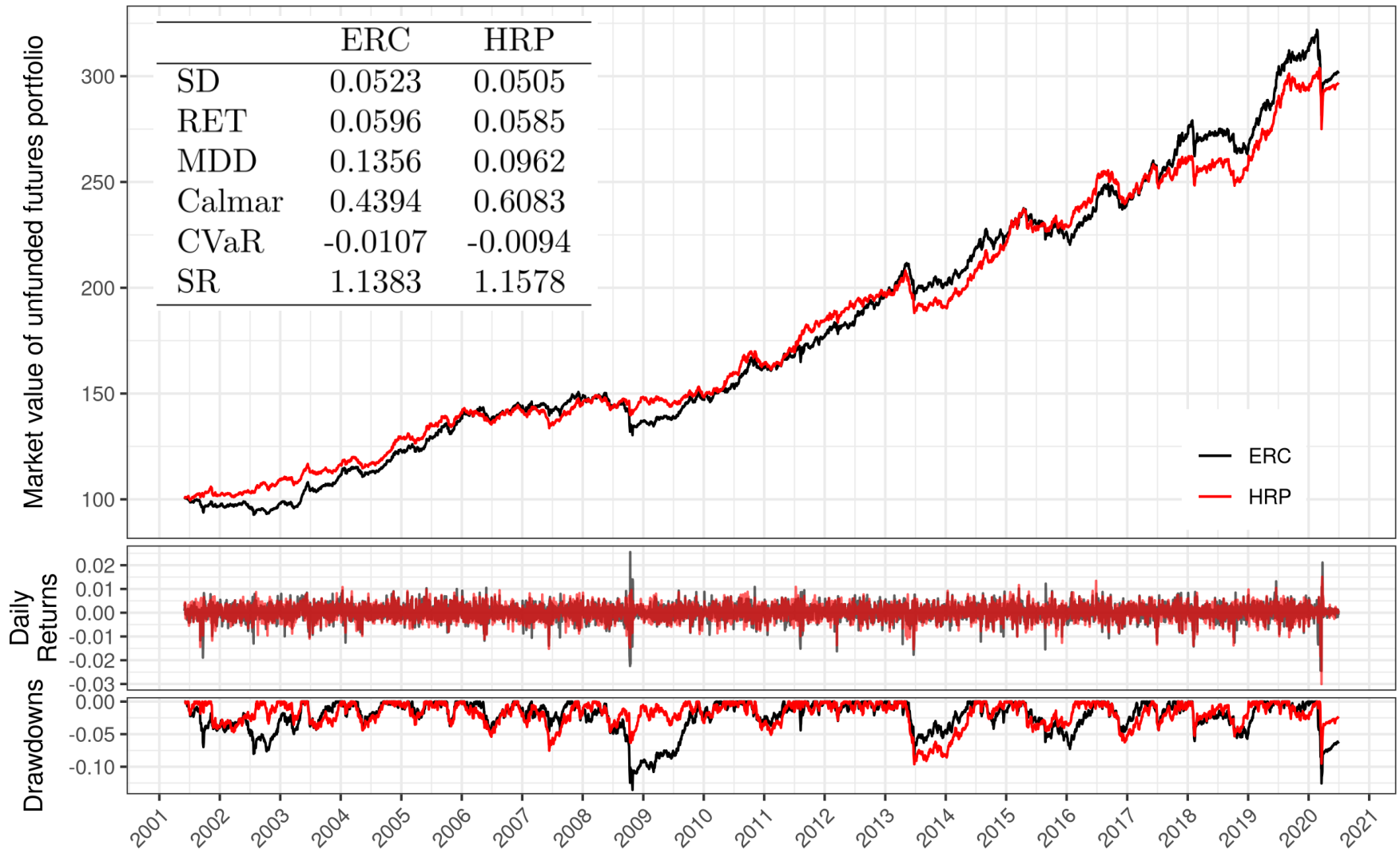
A practitioners' method to measure risk-adjusted returns:

$$\text{Calmar} := \frac{\text{annualized avg return}}{\text{worst drawdown}}$$

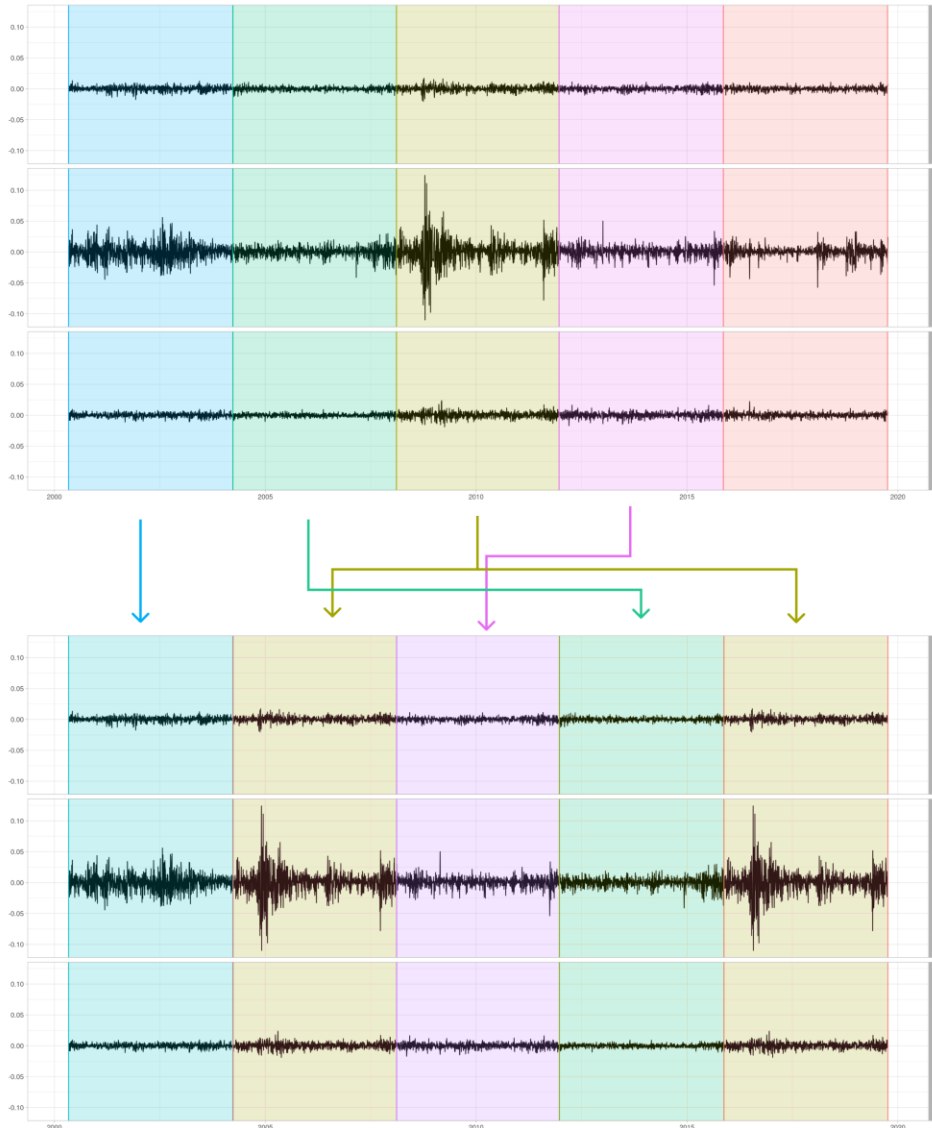
- Asymmetric: focus on the worst drawdown, not on volatility like in Sharpe ratio.
- Institutional investors often have to divest a specific strategy at a high drawdown (10%-15%).
- Inverse Calmar ratio: number of years it takes for the average return to make for the worst drawdown

MAR ratio: «Managed Account Reports», a 70's investment newsletter from Leon Rose  
Calmar ratio: Young, Terry W. (1991), "Calmar Ratio: A Smoother Tool", Futures magazine

# HRP shows less drawdowns in historical data. But elsewhere?

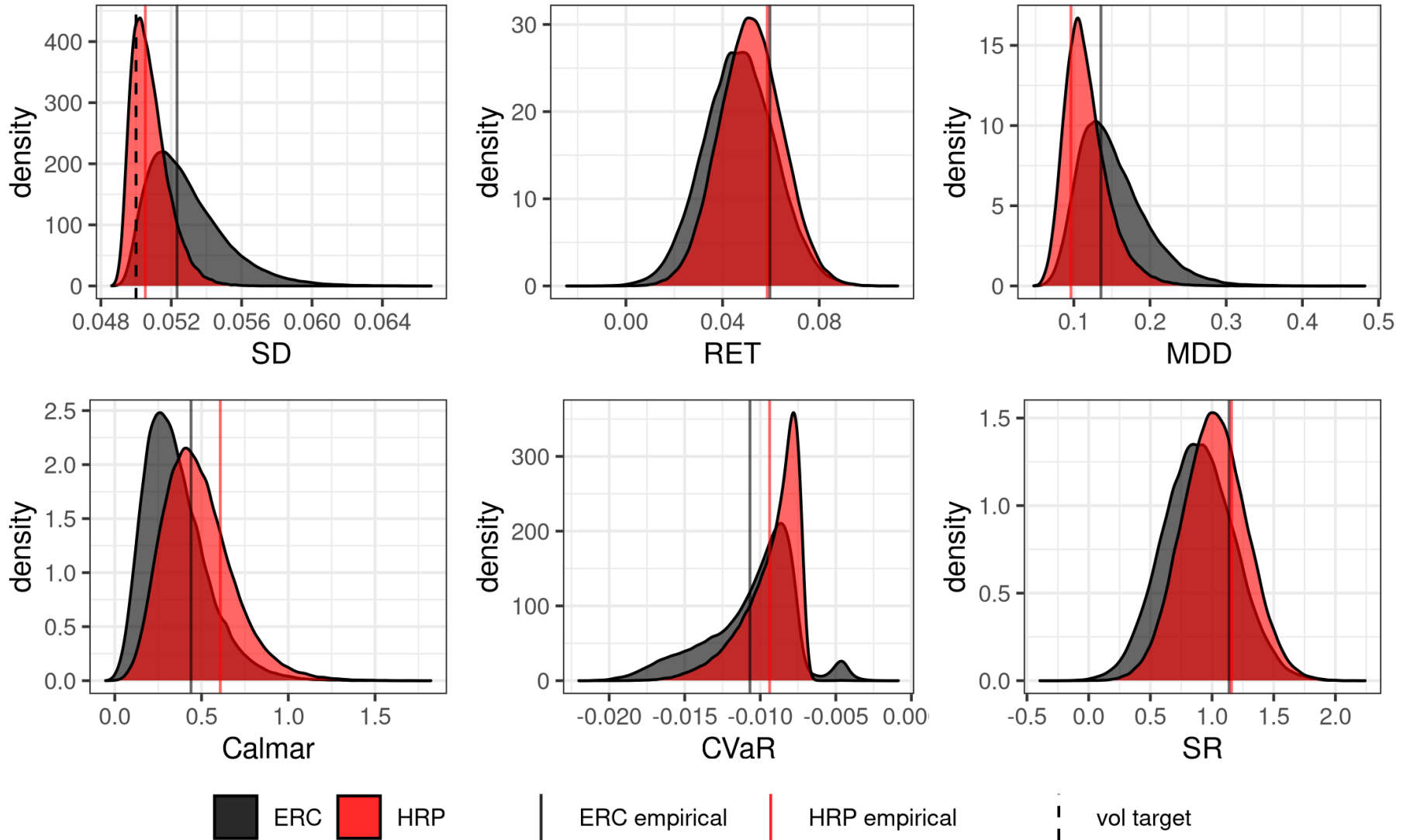


# Data augmentation - Bootstrap

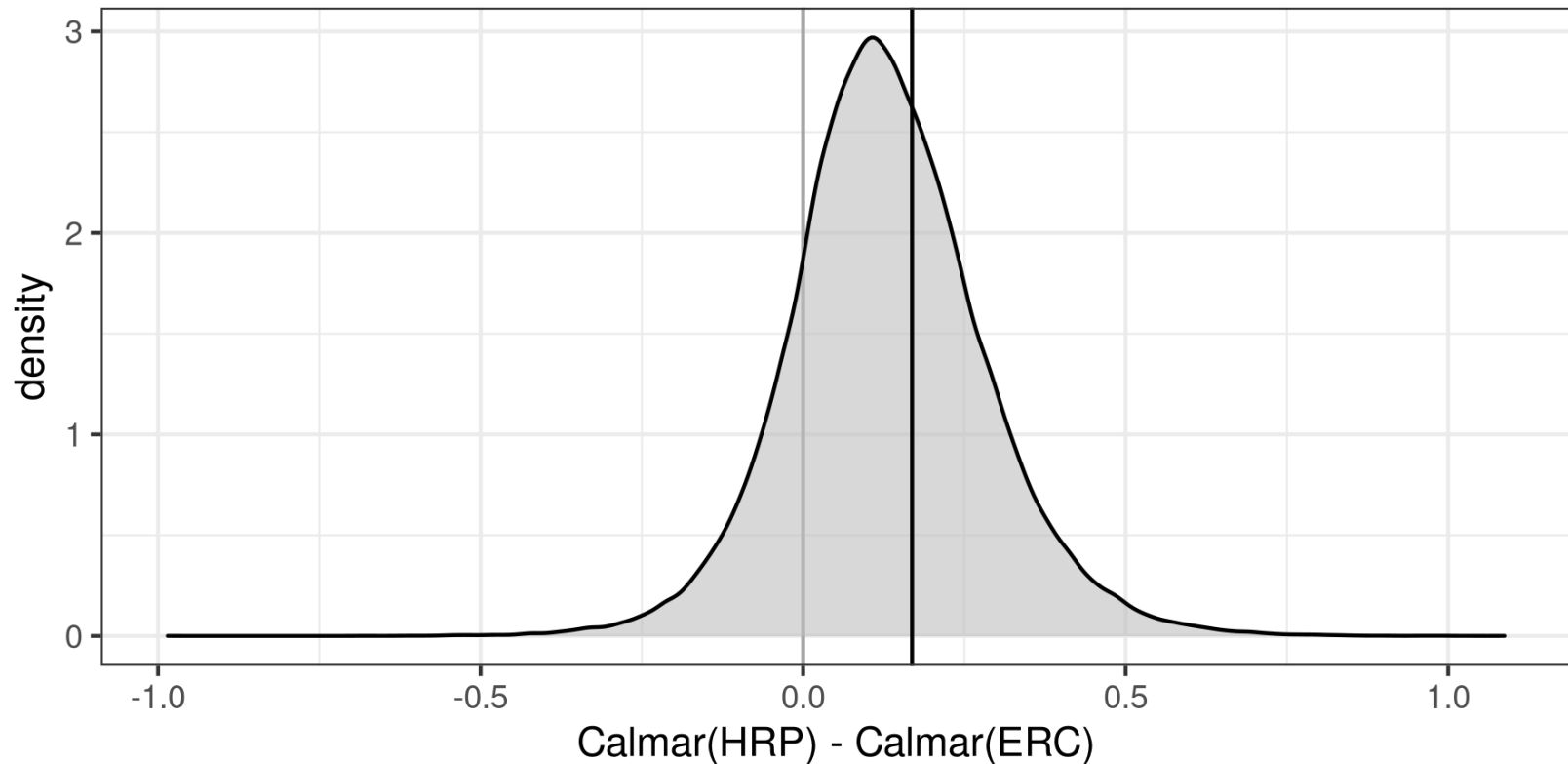


We want to test multiple scenarios:  
Block bootstrap  
blocks of 60 business days  
100k - portfolio universes

# Performances – 100k bootstrap portfolios



## What dataset features make HRP perform better than ERC?



We use **XGBoost**  $\leftrightarrow$  trained over a set of features (global properties of the portfolios), then we exploit Machine Learning explanations.

# 96 features: statistical properties of the bootstrapped portfolio

[ (sd.) ]      maxDD      ( \_futures )      \_mean  
 over time      statistical measure      asset class      aggregated

Traditional Measures  
40 x 2

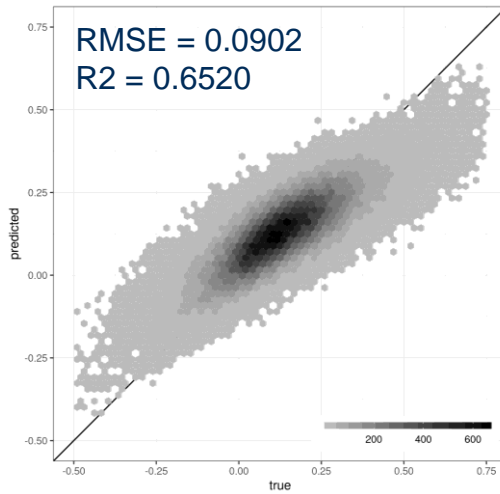
statistical measure			
mean			
mean			
sd		asset class	
standard deviation		futures	aggregated
corr		FixedIncome	_mean
correlation coefficients		Commodities	_sd
maxxDD		Equities	
maximum drawdown			
maxDDminus			
maxxDD of minus log-returns			

Clustering Measures  
2 x 4 x 2

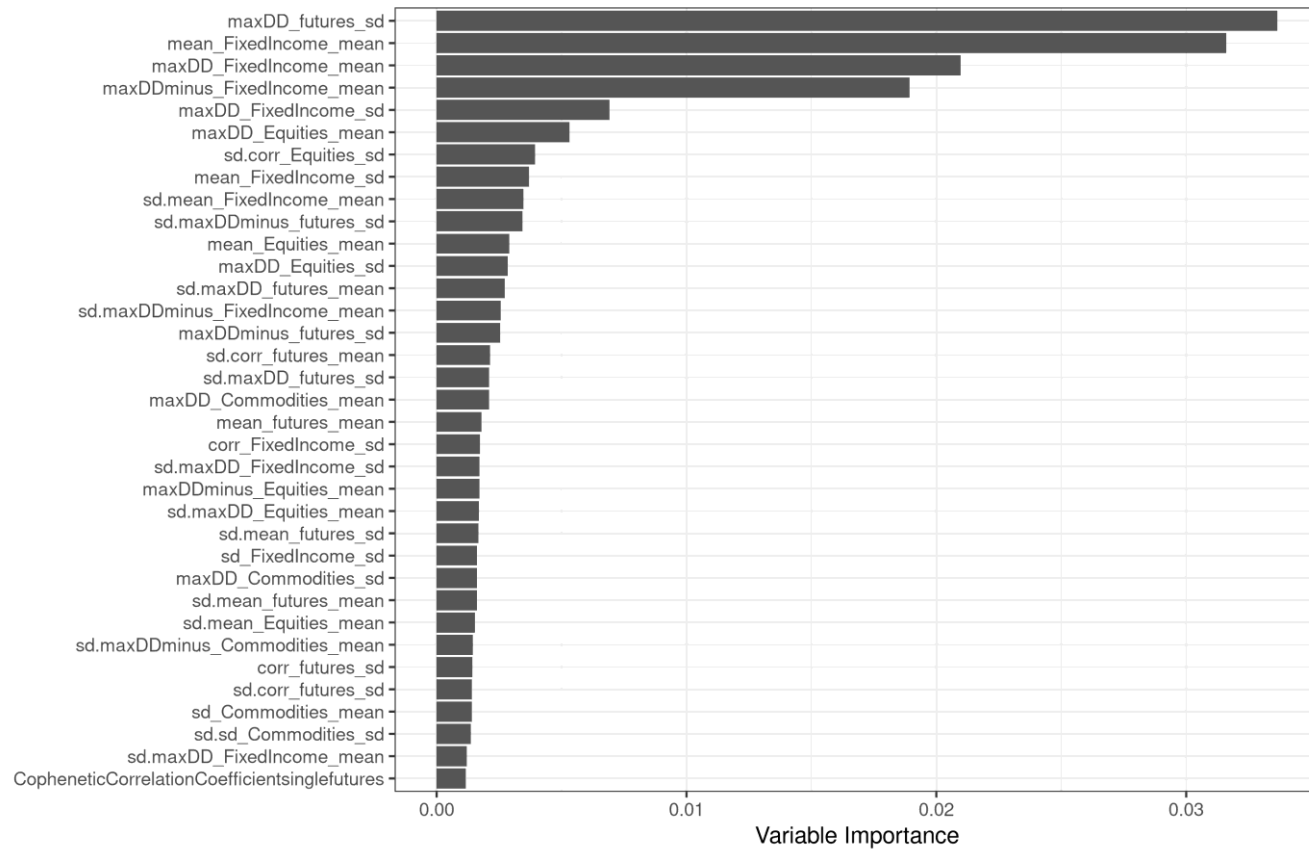
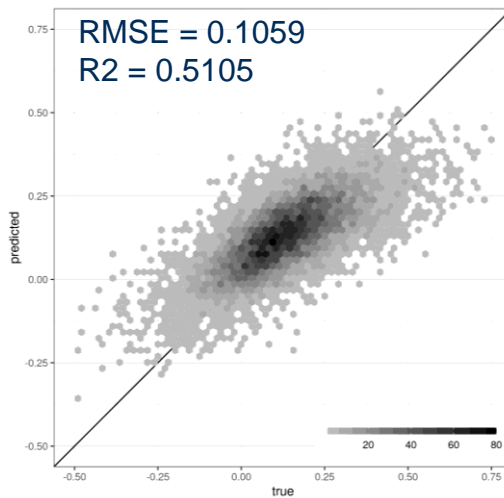
statistical measure		asset class
ClusterCoefficienttssingle	specifies the agglomerative coefficient as defined in Kaufman and Rousseeuw (2009) measuring the clustering structure of the dataset.	futures
CopheneticCorrelationCoefficientsingle	correlation between the distance matrix and the ultrametric distance matrix	FixedIncome
		Commodities
		Equities

# XGBoost outcomes

## Train

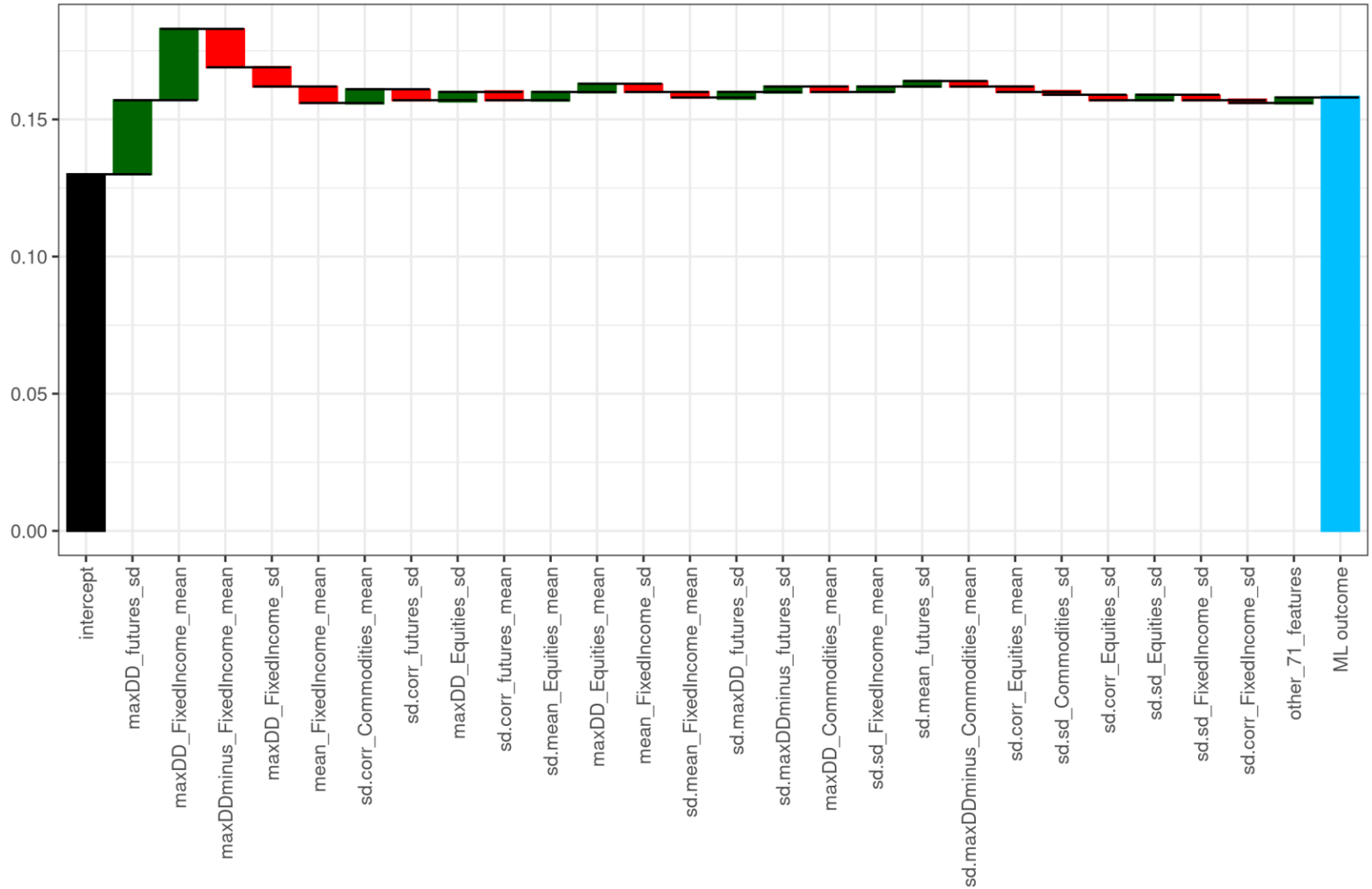


## Test

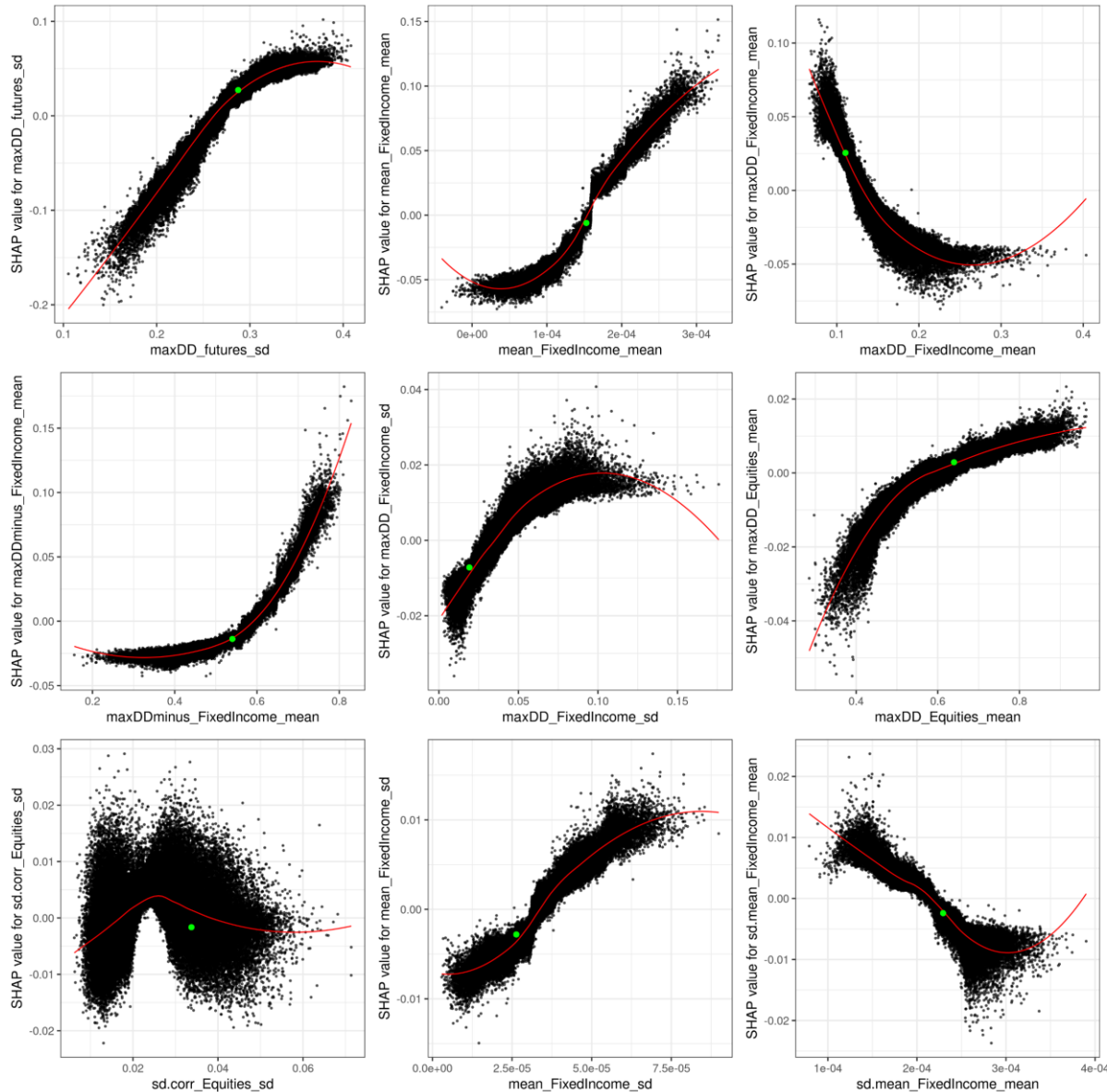




# SHAP Explanation for the empirical portfolio



# ML Explanations of the performance spread between HRP and ERC



# Conclusions and Extensions

- We find HRP (Hierarchical Risk Parity) to show less drawdowns than Equal Risk Contribution (ERC) using empirical and bootstrapped data.
- We train an explanation model to link the performances with features of bootstrapped market data.
- Out of sample, the Calmar ratio spread can primarily be explained by asset-specific measures of the bootstrapped dataset, especially by the fixed income performance.
- Extensions:

Matrix Evolutions: Synthetic Correlations and Explainable Machine Learning for Constructing Robust Investment Portfolios. <https://doi.org/10.3905/jfds.2021.1.056>

'Adaptive Seriation Risk Parity' and other Extensions for Heuristic Portfolio Construction using Machine Learning and Graph Theory, <https://dx.doi.org/10.2139/ssrn.3806714>

## Appendix: Risk Parity Portfolio

- 17 futures
- 3 asset classes
  - Fixed income
  - Equities
  - Commodities
- Time-series  
from 2000-05-03  
to 2020-06-30  
with daily prices

Ticker	Asset class	Currency	Name
CLA Comdty	Commodities	USD	NYMEX WTI Light Sweet Crude Oil
GCA Comdty	Commodities	USD	COMEX Gold
SIA Comdty	Commodities	USD	COMEX Silver
BZA Index	Equities	BRL	BM&F IBOVERSPA
ESA Index	Equities	USD	CME E-mini S&P 500
HIA Index	Equities	HKD	HKFE Hang Seng
NKA Index	Equities	JPY	OSE Nikkei 225
NQA Index	Equities	USD	CME E-mini NASDAQ-100
SMA Index	Equities	CHF	Eurex SMI
VGA Index	Equities	EUR	Eurex EURO STOXX 50
XPA Index	Equities	AUD	ASX SPI 200
Z A Index	Equities	GBP	ICE FTSE 100
CNA Comdty	Fixed Income	CAD	10Y Canadian GB
G A Comdty	Fixed Income	GBP	ICE Long Gilt
RXA Comdty	Fixed Income	EUR	Eurex 10Y Euro-Bund
TYA Comdty	Fixed Income	USD	CBOT 10Y US T-Note
XMA Comdty	Fixed Income	AUD	ASX 10Y Australian T-Bonds